# Evanescent wave coupling in terahertz waveguide arrays 

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#### Abstract

We study energy transfer among an array of identical finitewidth parallel-plate waveguides in close proximity, via evanescent wave coupling of broadband terahertz waves. We observe stronger coupling with larger plate separations and longer propagation paths. This work establishes a platform to investigate new opportunities for THz components and devices based on evanescent wave coupling.


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## 1. Introduction and background

The phenomenon of evanescent wave coupling in waveguides and waveguide arrays has been well studied in the visible and infrared and is an important concept underlying key technologies such as optical fiber splitters and combiners [1,2]. For example, evanescent wave coupling enables a directional coupler to function based on the spatial overlap of the evanescent field of one fiber with the mode profile of another fiber in close proximity [3]. Although evanescent coupling is very well developed in optics, and also in the microwave community [4,5], it has only recently begun to be studied in the terahertz ( THz ) range on structures such as two wire waveguides [6], porous fibers [7], photonic crystal fibers [8], parallel-plate dielectric waveguides [9], and rectangular waveguides [10]. Another common waveguide platform for broadband terahertz radiation is the parallel-plate waveguide (PPWG) [11]. This geometry is often employed because of the relatively low loss and the possibility of dispersionless operation via the fundamental transverse-electromagnetic (TEM) mode of the waveguide [12]. Although in the ideal PPWG, the plate width is infinite, recent work using finite-width PPWGs have demonstrated similar TEM-like behavior [13-15]. In light of this, we study broadband evanescent coupling between adjacent finite-width PPWGs, which has not previously been considered.

To explore waveguide coupling in the THz regime, we consider an array of finite-width PPWGs running parallel to each other. Finite-width PPWGs can exhibit energy leakage when the plate separation is large [13,16], suggesting that an array of finite-width PPWGs is a convenient platform for studying THz energy coupling between waveguides. With broadband single-cycle pulses and coherent detection of the THz field, extending evanescent wave coupling to the THz regime may enable interesting new applications in THz waveguide devices and components such as splitters and power combiners.

## 2. Methods

To observe THz evanescent wave coupling, we construct several finite-width PPWGs in the form of an array [Fig. 1(a)]. Each array consists of seven PPWGs. We choose a plate width $(w)$ of 2 mm , on the same order of the relevant wavelength, in order to ensure a strong evanescent field transverse to the waveguide axis. We fix the gap $(g)$ between plates of 2 mm to allow spatial overlap of the modes between waveguides. For ease of assembly, we fabricate the arrays by machining two solid aluminum pieces to have a series of corresponding "fins", one set comprising the top plate of each PPWG and the other set comprising the bottom plate [Fig. 1(b)]. By varying the distance between the top and bottom sets of the PPWG array, we control the plate separation (b) using precision micrometer mounts. We fabricate arrays of various propagation lengths $(L)$ ranging from 5 to 50 cm . We choose the depth (d) from waveguide edge to aluminum back plate of 18 mm so as to be large enough that no reflections occur from the aluminum; therefore, this back plate serves as a structural support only, and can be ignored from the point of view of the propagation of THz radiation in the waveguides.

To measure coupling across the array, we measure the field emerging from the output of the waveguide array using a commercial THz time-domain spectroscopy (THz-TDS) system with fiber-coupled photoconductive antennas [17]. The beam is vertically polarized to dominantly excite the TEM mode [11]. From the transmitter antenna, the light is focused using a pair of lenses in a confocal configuration and an aperture is used at the input face to ensure the incident THz beam excites only the central waveguide of the array. At the output, a 0.8 mm diameter aperture is placed in front of the receiver to improve the spatial resolution. We measure a time-domain waveform at 57 different locations along a line bisecting the two metal plates perpendicular to the propagation axis, shown by the dashed line in Fig. 1(a). This spatially resolved measurement is repeated for each plate separation and propagation length. Frequency-dependent spatial profiles are then obtained by Fourier transform of the measured time-domain waveforms.


Fig. 1. Design of finite-width PPWG array with the orange circle illustrating the THz beam spot on the input facet of the central waveguide. (a) Transverse cross section of array with dimensions: width $w=2 \mathrm{~mm}$, gap $g=2 \mathrm{~mm}$, and variable plate separation $b$. The output field is measured along the horizontal dashed line in steps of 0.5 mm . (b) Experimental design of array from two machined Al plates showing length $L$ in the propagation direction and depth $d$ $=18 \mathrm{~mm}$ to Al back plate. Arrays with various values of $L=5,10,15,20,25,30,40$ and 50 cm are studied.

Using this procedure, we obtain the electric field distribution across the array as a function of $b, L$, and frequency. Typical results are shown in Fig. 2(a) which depicts the measured electric field as a function of frequency (vertical axes) and horizontal position along the waveguide array (horizontal axes), for a propagation length $L$ of 30 cm . The four panels show the results for four different plate separations. Assuming each waveguide supports a mode of approximately Gaussian shape along the x-direction [15], for a selected frequency we fit Gaussian curves centered at each waveguide [Fig. 2(b)] using the power normalized to the maximum value at the center of the array. In the fitting, we constrain the widths and centroids of the Gaussian curves to be the same for all waveguides, for a particular plate separation while allowing the fitting algorithm to optimize the amplitude. Therefore the number of free parameters in the fitting procedure is only one per waveguide. This analysis yields the total power contained in each waveguide at the output aperture of the array.


Fig. 2. (a) Electric field maps of frequency versus x-position across array. Red dashed line indicates frequency slice analyzed in Fig. 1(b). Each frame shows frequencies of $10-300 \mathrm{GHz}$ and is normalized to maximum value. The peak in the spectral amplitude at $\sim 150 \mathrm{GHz}$ reflects the spectral content of the input pulse. (b) Frequency slice at $f=125 \mathrm{GHz}$ of $L=30 \mathrm{~cm}, b=1$ mm device showing experimental data (black line) and Gaussian fits (red lines). The vertical grey bars indicate the locations of waveguides in the array.

Our experimental results are compared to the results of finite element method (FEM) simulations using Comsol Multiphysics [18]. We exploit the symmetry of the problem to model only one quarter of the waveguide array, using perfect-electric-conductor (PEC) boundary conditions horizontally (along the x-direction) and perfect-magnetic-conductor
(PMC) boundary conditions vertically (along the y-direction) on the inside boundaries [19]. Perfectly matched layers (PML) surround the model to absorb radiation at external boundaries without reflections. We model the edges of waveguides using PEC boundary conditions, rather than as real metals, to conserve computing resources. At a particular frequency, plate separation, and length, the resultant electric field is extracted along the output of the array for comparison to experimental results. FEM simulation is an important tool here, since there is no closed-form analytical solution to describe the modes of a finite-width PPWG.

## 3. Results

In order to characterize the evanescent wave coupling between THz finite-width PPWGs, we study the dependence of our results on two specific variables: plate separation (b) and propagation length $(L)$. The plate separation directly effects the energy confinement within each waveguide and the propagation length influences the cumulative degree of interaction between waveguides. In order to quantify our results, using the Gaussian curve fitting described above, we determine the area under these curves, which describes the power contained in a particular waveguide. Assuming that the amount of power that couples from the central waveguide to the first most adjacent waveguides is the same as the amount of power that couples from the first most adjacent waveguides to the second most adjacent, we compute the ratio of powers between all pairs of adjacent waveguides in the array. We define the power ratio as the average of these results for all plate separations and propagation lengths. Figure 3 shows the values of these averaged power ratios as a function of $b$ and $L$ for a representative frequency.

Using finite-element-method (FEM) simulations, we also numerically computed the solution for a given geometry and applied the same analysis of Gaussian fitting to this computational result. Within the range of lengths for which the simulations are feasible (given finite computing resources, $L \leq 20 \mathrm{~cm}$ ), we see good agreement of the power ratios between the experimental results (solid lines) and the numerical simulations (dashed lines), which indicates the robustness and reliability of the experimental results.


Fig. 3. Power ratios extracted from experimental data (data points and solid lines) as described in the text, along with FEM simulations (dashed lines). Stronger coupling is observed for larger plate separations and longer propagation path lengths.
As mentioned, the plate separation (b) has a significant influence on the energy confinement within each waveguide. As is evident from Fig. 2(a), greater plate separations yield more energy distribution across the array, which is evidence of greater coupling from the central waveguide to adjacent waveguides. In finite-width PPWGs, energy confinement
decreases as plate separation increases [13]. Thus, when the energy is less confined, the mode profile is broader and there is more evanescent leakage from the open sides of the waveguide. The evanescent energy then overlaps into adjacent waveguides and the effect continues spreading energy across the array.

The confinement mechanism can be attributed to plasmonic edge effects. Although surface plasmons are not supported by a flat metal surface, plasmons can be supported by a structured or non-planar metal interface. These non-planar plasmonic effects indeed have been seen at optical frequencies in slot waveguides [20] and recently, evidence of plasmonic edge modes have been observed in finite-width PPWGs in the THz regime [13-15,19]. Since metals have a high conductivity in the terahertz range, the difference between an edge plasmon on a PEC and an edge plasmon on a real metal is small. Thus, in our FEM simulation using PEC for waveguide boundary conditions, we see evidence of this edge plasmon confining mechanism.


Fig. 4. Full-width-at-half-maximum (FWHM) of Gaussian fits to experimental data at $f=125$ GHz where the average FWHM is displayed for each plate separation. From lowest to highest plate separation, the FWHM increases as plate separation increases, indicating a change of the spatial intensity profile. The trend with $b$ is clear; however, variations in FWHM with propagation length may be due to alignment issues or other experimental uncertainty.

In the PPWG array, when the plates are closer together, the plasmonic edge modes couple more strongly to each other yielding a higher energy confinement within the waveguide [15]. Higher energy confinement implies that the evanescent tails of the propagating mode profile is less pronounced, and thus the apparent mode width is smaller. Since the width of each Gaussian fit to the field-intensity profile is fixed for each plate separation and propagation length, the full-width-at-half-maximum (FWHM) can be extracted as a function of these two variables, as shown in Fig. 4. Indeed, we observe a trend of smaller mode widths for plates that are closer together, a result that is consistent with the aforementioned phenomenon of plasmonic energy confinement in finite-width PPWGs.

The propagation length $(L)$ also influences the spreading of energy across the waveguide array. Similar to the situation well known from fiber optics, as the propagation length is increased, the effective interaction length between waveguides is increased [3]. Figure 5 shows how energy spreads across the array for increasing propagation length. As the THz signal propagates for longer distances in a PPWG, the energy loss out of the open sides of the waveguide increases $[11,16]$. In the design of our array, the leaked energy of the central excited waveguide spills out into the adjacent waveguides on either side; then, this energy in
turn spills into the next most adjacent waveguides. As in the case of evanescent wave coupling between two adjacent fibers [21,22], we anticipate that the degree of energy coupling between adjacent PPWGs should grow exponentially with propagation length.


Fig. 5. Electric field maps of frequency for various propagation lengths $(L)$ at a fixed plate separation $b=1 \mathrm{~mm}$, each frame showing frequencies of $10-300 \mathrm{GHz}$. Each plot is normalized to its maximum value. As in Fig. 2(a), the spectral maximum at about 150 GHz corresponds to the peak of the spectrum of the input pulse.

With this assumption, we extract a coupling coefficient at each wavelength by fitting the measured power ratios [e.g., Fig. 3] to an exponential function the form $y=e^{c_{x}}-1$ where $C$ is the coupling coefficient, and $x$ represents propagation distance. From fits such as this, we extract the values of $C$ as a function of wavelength and plate separation. Figure 6 shows these results for $C(\lambda)$, for several values of the plate separation. This result indicates that the coupling coefficient is approximately independent of wavelength, over the measured range.


Fig. 6. Coupling coefficient (C) versus wavelength for given plate separations. Shaded regions demark boarders of operational frequencies from about $50 \mathrm{GHz}(6 \mathrm{~mm})$ to $200 \mathrm{GHz}(1.5 \mathrm{~mm})$.

This weak wavelength-dependence of $C(\lambda)$ suggests that the confinement mechanism for the guided mode is not strongly wavelength-dependent, and therefore, cannot depend critically on the plate width. (We note that the plate width $w=2 \mathrm{~mm}$ corresponds to the range
from $\sim 0.3 \lambda$ to $\sim 1.3 \lambda$ within the measurement bandwidth.) Instead, this result is consistent with the aforementioned description in which the confinement mechanism is mediated by the excitation of edge plasmons at the waveguide's corners, as discussed previously [13]. We also observe that the coupling coefficient depends on the plate separation $b$ in a non-linear fashion. This is not surprising, since the energy confinement varies in a non-linear fashion with respect to $b$ as observed previously [13].

## 4. Conclusion

Through this design of a PPWG array, we have seen evidence of evanescent wave coupling in THz finite-width PPWG arrays. By varying plate separation and propagation length, we have investigated factors that influence this coupling. We have found the relationship that greater plate separation and longer propagation path lengths lead to stronger coupling. These results were consistent with numerical results from FEM simulation, and with the idea that excitation of edge plasmons mediates the energy confinement in finite-width PPWGs in the THz range. This work establishes a platform to investigate new opportunities for THz devices based on evanescent coupling.

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